NASA Technical Paper 3654

Prelude to Cycle 23: The Case for a Fast-Rising, Large Amplitude Cycle

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I. INTRODUCTION

Predicting the size and shape of a sunspot cycle continues to be an important issue, at least once every 11 years or so. It is an especially difficult task when very little, if any, of the unknown cycle has been revealed. Still, from a spacecraft missions point of view, it is imperative that we have an inkling of when conventional onset should occur, of how big the cycle might be, and of what the shape of the cycle may look like. Often, the first step in the process of answering these questions is to examine the statistical aspects of sunspot cycles. Such an approach, however, has only limited value. A second approach is to examine the data in light of possibly significant secular trends. A third way uses phenomena or information from the previous cycle to gauge the relative size and shape of the present cycle (in this case the previous cycle would be cycle 22 and the present cycle would be 23). In some respects, this approach is similar to a fourth approach that uses precursors (measurements of parameters early in the rise of the cycle that may be related to size and shape constraints on the cycle). Brown and Simon[1] have discussed the limitations of each of these techniques as they were applied to the end of cycle 21 and the onset of cycle 22 in order to effect a prediction for cycle 22 (in particular, its size).

In this paper, we continue our study of the decline of cycle 22 and what this seems to suggest about the anticipated size and shape of cycle 23. Recently (Wilson et al.[2]), we presented evidence that the decline of cycle 22 is consistent with the notion that it will have a length shorter than 11 years, making it a short-period cycle (cf. Rabin et al.,[3] Wilson[4]). If true, cycle 22 will end and cycle 23 will begin (conventional onset) about December 1996 (±3 months). We noted that, as of early 1996 (i.e., April 1996), no new cycle, high-latitude (≥25˚) spots had been observed. This is important since the occurrence of a new cycle, high-latitude spot has always preceded conventional onset of a sunspot cycle by at least 3 months (11 out of 11 cycles); hence, we inferred that onset for cycle 23 had not already occurred, but lay ahead in the near future. (Please note that beginning first in mid May 1996, new cycle, high-latitude spots have begun to appear, suggesting that conventional onset for cycle 23 is now expected to occur anytime after about July 1996.)

In a second paper (Wilson et al.[5]), we noted that several statistically important associations are apparent in the sunspot record, associations that have a distinct bearing on the expected size and shape of cycle 23. For example, we described an amplitude-period relationship (cf. Hathaway et al.[6]), which shows that when the previous cycle is of shorter (longer) than average length, the following cycle usually is larger (smaller) than average size. We also described the Waldmeier effect, which states that cycles of shorter (longer) than average ascent duration usually are larger (smaller) than average size. Additionally, we spoke of the maximum-minimum effect, which shows that cycles of larger (smaller) than average minimum amplitude usually are of larger (smaller) than average maximum amplitude, and of the even-odd cycle effect, which shows that when we group cycles according to cycle numberedness, the preferred grouping is even-leading-odd-following and that the odd-following cycle usually has been the larger of the two cycles (cf., Kopecký,[7] Wilson[8]). The implication of this study was that cycle 23 is expected to be a large-amplitude, fast-rising cycle, with maximum amplitude expected sometime between July 1999 and October 2000.

Our most recent study (Wilson et al.[9]) examined more closely the statistical aspects and interrelationships of rise time, size, and duration of sunspot cycles. In particular, we showed that the three sunspot cycle-related parameters of ascent duration, maximum amplitude, and period are not mutually independent. Instead, associated aspects exist that assist in the classification of sunspot cycles according to the value of these parameters. We also showed that within the first 12 months of the sunspot cycle, we usually can identify the correct rise-amplitude class of the cycle (postdicting correctly 19 of 21 cycles). The criterion is whether or not a smoothed sunspot number is above or below the mean curve for a particular elapsed time t from cycle minimum (onset). For cycle 22, we showed early-on that it probably would be a fast-rising, large-amplitude cycle, which turned out to be true; we also suggested that it probably will be a short-period cycle (having a period shorter than 11 years—about 123±3 months in length), suggesting an onset date of about December 1996±3 months for cycle 23, something that has yet to be confirmed. So, by close monitoring of the smoothed sunspot number curve during the rising phase of cycle 23, we should be able to gauge whether or not cycle 23, indeed, will be a fast-rising, large-amplitude, short-period cycle or, instead, be a slow-rising, small-amplitude, long-period cycle.

In this investigation, we present the case for cycle 23 to be a fast-rising, large-amplitude cycle, which probably will also be of short-period. We do this based on inferred secular trends in three particular data sets for the common data-available interval of cycles 12 to 22 (from about 1878). These data sets include annual averages of minimum amplitude (R(min)) and maximum amplitude (R(max)) of sunspot number and the minimum amplitude of the aa geomagnetic index in the vicinity of sunspot minimum (aa(min)). Now, prior to the start of conventional onset of cycle 23, we believe that the year of minimum amplitude for cycle 23 will be 1996; that R(min) will have a value between 7 and 18; that aa(min) will occur either in 1996 or, more likely, in 1997, having a value between 16 and 22, probably >19.0 based upon the even-odd cycle effect; that the year of sunspot maximum probably will be 1999, although it could be 2000; and that R(max) ≥114.9 and quite possibly >157.6 based on the even-odd cycle effect.

II. RESULTS

Figure 1 shows the cycle-by-cycle variation of R(min), R(max), and aa(min) for the common data-available interval of cycles 12 to 22, dictated by the aa index. These annually averaged data were taken from Wilson[10] and Wilson et al.[5] While the distribution of R(min) values may be due entirely to chance, based upon the results of Fisher's exact test (Everitt,[11] p. 15) for the observed 2 by 2 contingency table (determined by the median values of the individual parameters), or one more suggestive of a departure from independence (chance), since P >5 percent, the distributions of aa(min) and R(max) are, in contrast, strikingly systematic in appearance. The probability P of obtaining the observed distributions, or ones more suggestive of a departure from independence, for these parameters is 0.2 percent, a statistically significant result. Supporting the view that both aa(min) and R(max) are nonrandomly distributed is the result of a runs test (Langley,[12] p. 322) on each parameter (not shown). Notice that for both R(max) and aa(min), the number of cycles with values below the median is 5 and the number equal to or above the median is 6, consisting of exactly 2 runs. The probability of this occurring by chance is ≤5 percent. Likewise, for all three of the parameters, hypothesis testing of the difference of the two means for the independent samples of cycles 12 to 16 and 17 to 22 (Lapin,[13] p. 486) reveals that the means for the two samples are significantly different (at the 98-percent level of confidence, *cl*), with the latter group of cycles having the larger means. Each parameter can be fitted by a linear regression (shown as the diagonal line in each panel) based on standard linear regression analysis. In each case, the inferred regression is statistically significant at *cl* >99 percent. Thus, cycles 17 to 22, the most recent of sunspot cycles, clearly have been associated with enhanced sunspot number minima and maxima and with enhanced geomagnetic activity in the vicinity of cycle minima.

In figure 1, each of the inferred regressions has a correlation coefficient $r \geq 0.79$, suggesting that about two-thirds or more of the variance in each of the parameters can be explained by the linear fits. Extrapolating these inferred secular increases of parametric value with time allows us to compute the expected value for each of the parameters for cycle 23; namely, we find that, for cycle 23, R(min) = 12.7±5.7, aa(min) = 21.0±5.0, and R(max) = 176.7±61.8, where the range for each of the parameters represents the 95-percent confidence level for the prediction (computed as 2.262 times the standard error of estimate se for the given sample size). Thus, there is only a 2.5-percent chance that either R(min) <7.0 or R(min) >18.4. Similarly, there is only a 2.5-percent chance that aa(min) <16.0 or aa(min) >27.0 and that R(max) <114.9 or R(max) >238.5.

Figure 2 depicts the frequency of occurrence histograms for aa(min) and R(max) occurrences relative to R(min) occurrence. Notice that for 8 of 11 cycles, aa(min) followed R(min) by 1 year, and for only 3 of 11 did aa(min) and R(min) occur in the same year (these include cycles 14, 15, and 19). On the other hand, for 4 of 11 cycles, R(max) followed R(min) by 3 years (these include cycles 18, 19, 21, and 22); for 5 of 11, R(max) followed R(min) by 4 years (these include cycles 13, 14, 15, 17, and 20); and for 2 of 11, R(max) followed R(min) by 5 years (these include cycles 12 and 16). Thus, any formal prediction of R(max) that is based in part or whole on the observed value of aa(min) cannot be made until the rising portion of the cycle is already clearly underway (these include Ohl's method, Kane's method, and Wilson's bivariate technique; see Wilson[10]).

Figure 3 displays the even-odd cycle effect for the three parameters. Of the three, only the fits for R(max) and aa(min) seem to be statistically important. The even-odd cycle effect suggests that cycles are grouped in pairs, with the preferred grouping being even-odd in that order. Thus, cycles 22 and 23 represent a new cycle pairing and, because we already know the values of R(max) and aa(min) for cycle 22 (equal to 157.6 and 19.0, respectively), we can infer their values for cycle 23; namely, R(max) = 202.1±35.0 and aa(min) = 22.1±7.0 (at the 95-percent level of confidence). Thus, cycle 23 may very well have R(max) ≥167.1 and aa(min) ≥15.1.

Figure 4 shows the correlational aspects of R(max) versus aa(min) and of aa(min) and R(max) versus R(min). For the R(max) versus aa(min) fit, the coefficient of correlation $r = 0.90$, implying that about 82 percent of the variance in R(max) can be explained by the variation in aa(min). With its relatively small standard error of estimate se, one could easily use the observed value for aa(min) to estimate the size of the cycle about 2 to 4 years in advance of its actual occurrence (cf. Kane,[14 15] Ohl,[16] Ohl and Ohl,[17] and Wilson[10]).

Figure 5 plots the relative error between the observed R(max) and the predicted R(max) for cycles 12 to 22, based on the aforementioned regression fits (the secular fit, bottom panel; the maximum-minimum fit, middle panel; and the R(max) versus aa(min) correlational fit, top panel). The relative error is computed as a ratio: observed value divided by predicted value. For the secular fit, we find that 10 of 11 cycles have their observed R(max) within 30 percent of the predicted value. Thus, based on this fit, for cycle 23, R(max) = 176.7±53.0, or R(max) ≥123.7 and R(max) ≤229.7. For the maximum-minimum fit, we find that 8 of 11 cycles have had their observed R(max) to lie within 30 percent of their predicted R(max) values. Presuming R(min) = 12.7 (from the secular fit), the maximum-minimum fit suggests that cycle 23 should have R(max) = 147.9±44.4, inferring that R(max) ≥ 103.5 and R(max) ≤ 194.3. For the R(max) versus aa(min) correlational fit, we find that 9 of 11 cycles have had their observed R(max) to lie within 20 percent of their predicted R(max) values. Presuming aa(min) = 21.0 (from the secular fit), the R(max) versus aa(min) correlational fit suggests that cycle 23 should have R(max) = 178.7±35.7, inferring that R(max) ≥143.0 and R(max) ≤214.4. Thus, whether we base our decision on the secular fit, the maximum-minimum fit, or the R(max) versus aa(min) correlational fit, which is the strongest of the three, we are led to believe that cycle 23 probably will have an R(max) that is larger than average in size.

Figure 6 shows the relative error between observed R(max) and predicted R(max), based on the very small sample of even-odd cycle pairs. We find that for 5 of 5 cycle pairs the observed R(max) has been within 15 percent of the predicted R(max). Therefore, given that R(max)[22] = 157.6, we compute R(max)[23] = 202.1±30.3, or that R(max)[23] ≥171.8 and R(max)[23] ≤232.4, again, highly suggestive that R(max) for cycle 23 very likely will be above average in size, possibly, well above average in size.

Figure 7 displays the relative error between observed R(max) and predicted R(max), based on the bivariate fit of R(max) against R(min) and aa(min) (Ehrenberg,[18] p. 200). The fit has $r = 0.99$ and se = 8.2. For 9 of 11 cycles, the observed R(max) has been within 10 percent of the predicted R(max), inferring that, for cycle 23, we should expect R(max) = 175.1±17.5, using the secular-fit estimates of R(min) and aa(min) for cycle 23 (= 12.7 and 21.0, respectively). For 11 of 11 cycles, the observed R(max) has been within 15 percent of the predicted R(max). So, again, presuming the correctness of the secular-fit estimates of R(min) and aa(min) for cycle 23, we expect R(max) for cycle 23 to be equal to 175.1±26.3, or that it should have a value that lies between 148.8 and 201.4. Once R(min) and aa(min) have actually been observed, a more definitive prediction for R(max) can be made, probably in late 1997 at the earliest.

III. DISCUSSION AND CONCLUSIONS

The preceding section has shown that a long-term, linear, secular increase is embedded in each of the records of R(min), R(max), and aa(min). Extrapolating these fits to cycle 23 allows us to estimate the sizes of these parameters before its conventional onset has even occurred. Thus, we found that, at the 95-percent level of confidence, R(min) ≥7.0, R(max) ≥114.9, and aa(min) ≥16.0 for cycle 23. Although based on a very small sample size (5 cycle pairs), the data also suggest that an even-odd cycle effect is apparent for both R(max) and aa(min), one in which the odd-following cycle is numerically larger than the even-leading cycle. Because R(max) and aa(min) for cycle 22 are already known (= 157.6 and 19.0, respectively), we infer from the even-odd cycle effect that, R(max) ≥167.1 and aa(min) ≥15.1 for cycle 23. Furthermore, the common behavior of R(min), R(max), and aa(min) suggests that they are not mutually independent. Indeed, regression analysis has revealed that the size of a sunspot cycle at maximum is directly related to the size of the cycle at minimum (the maximum-minimum fit) and, in particular, to the size of aa(min) (the R(max) versus aa(min) correlational fit). Comparison of observed values of R(max) to predicted values of R(max) using the various techniques has shown that, based on sunspot cycle number SCN (i.e., the secular fit), we can correctly infer the size of the cycle at maximum to within 30 percent (true for 10 of 11 cycles). Similarly, by waiting until R(min) has actually been observed, we also can correctly infer the size of the cycle at maximum to within 30 percent (true for 8 of 11 cycles). However, by waiting until after aa(min) has actually been observed (which usually follows R(min) occurrence by 1 year), we can correctly infer the size of the cycle at maximum to within 20 percent or, by using the bivariate technique, to within 15 percent (a value that also has been true for 5 of 5 even-odd cycle pairs, based on the size of the even-leading cycle). This demonstrates, then, that we can easily surmise the size of the sunspot cycle at maximum at least 2 to 4 years ahead of its actual occurrence. Having a reliable estimate for the size of the cycle and knowing both its minimum amplitude and conventional onset date, we also can apply curve-fitting algorithms (e.g., Hathaway et al.[6]) to the sunspot cycle curve to better estimate its shape. Because R(min) and aa(min) have yet to be observed for cycle 23, clearly, we cannot effect a formal prediction of R(max) for cycle 23 or of its shape until late 1997, at the earliest.

Figure 8 displays the values of the monthly means of the aa geomagnetic index (top panel) and of sunspot number (bottom panel) for January 1994 through early 1996. The annual averages for the parameters are drawn as horizontal lines. To the right of each is the expected interval range for R(min) and aa(min) for cycle 23, based on the secular fit. It is apparent that monthly mean sunspot number is now well within the interval of values that is indicative of R(min). So, it seems quite likely that R(min) will occur in 1996, having a value <18 (probably between 7.0 and 12.5) (cf. Fyodorov et al.[19]). Likewise, although aa(min) may very well occur in 1997, we cannot rule out that it might occur in 1996, having a value <22 (probably between 16 and 21). Presuming a value of about 10 for R(min) and about 20 for aa(min), at this stage of the cycle, perhaps a rather presumptuous act, we infer that R(max) for cycle 23 should be about 181.0±27.2, based on the bivariate technique, which can be compared to a value of about 170.8±34.2, based strictly on the anticipated aa(min) value alone.

The above values for R(max) are in close agreement to that predicted by the secular fit and the lower portion of the prediction interval of the even-odd cycle fit. They are within the upper-portions of the prediction intervals of the amplitude-period fit (Wilson et al.[5]), presuming that cycle 22 is a short-period cycle of length equal to 123 months, and of the SODA (i.e., Solar Dynamo Amplitude; cf. Schatten and Pesnell[20]) index fit (Schatten et al.[21]). Figure 9 compares the various predictions of maximum amplitude for cycle 23 that have been presented here and in Wilson et al.[5] The range of overlap of these 95-percent level of confidence predictions is about 167 to 200, inferring that cycle 23 probably will be the second largest cycle on record or, perhaps, smaller if R(min) and aa(min) turn out to be smaller than what their secular fits suggest for cycle 23. In any event, plainly, it is obvious that cycle 23 is expected to be larger than average in size; consequently, by the Waldmeier effect, we infer that it very probably will be a fast riser. Presuming the occurrence of R(min) in 1996, we expect R(max) to occur either in the year 1999 or 2000, with smoothed sunspot number maximum probably occurring within the year of maximum amplitude occurrence. Presuming smoothed sunspot number minimum occurring about December 1996 (±3 months) (cf. Wilson et al.[5]) and that cycle 23, indeed, will be a large-amplitude, fast-rising cycle, we infer that smoothed sunspot number maximum for cycle 23 should occur <48 months after minimum occurrence, probably about 41±7 months (the average ascent duration for a fast-rising cycle) after minimum. This suggests that conventional maximum should be expected about May 2000 (±10 months), or after July 1999.

In closing, this study has presented the case for cycle 23 to be a fast-rising, large-amplitude sunspot cycle, one that may rival the size of cycle 19. The bases for this conclusion rely on the existence of long-term, linear, secular increases within three particular data sets—R(min), R(max), and aa(min); on the existence of an even-odd cycle effect in the R(max) and aa(min) data sets, one that suggests that cycles are preferentially grouped as cycle pairs (even-odd, in that order), with odd-following cycles being the stronger of the two; on the existence of strong correlational behavior between the data sets (the maximum-minimum fit, the R(max) versus aa(min) correlational fit, and the bivariate fit); and on the findings of others using different techniques (the amplitude-period effect [20] and the SODA index[21]). Thus, because the *International Space Station* will begin its on-orbit construction in late 1997 (to continue for several years), it will be ongoing during the volatile rising phase (through maximum phase) of cycle 23. Because the *International Space Station* will be placed in a high-inclination orbit, space mission planners must pay especially close attention to the rapidly changing solar activity and geomagnetic activity levels that will be experienced during this time, in that enhanced activity suggests increased drag and higher than expected radiation dosage, if, indeed, cycle 23 is of near record size.

REFERENCES

1. Brown, G.M., and Simon, P.A.: in “Solar-Terrestrial Predictions: Proceedings of a workshop at Meudon, France, June 18–22, 1984.” P.A. Simon, G. Heckman, and M.A. Shea (eds.), NOAA, Boulder, CO, 1986, p. 1.

2. Wilson, R.M., Hathaway, D.H., and Reichmann, E.J.: J. Geophys. Res., vol. 101, No. A9, 1996, p. 19967.

3. Rabin, D., Wilson, R.M., and Moore, R.L.: Geophys. Res. Lett., vol. 13, No. 4, 1986, p. 352.

4. Wilson, R.M.: J. Geophys. Res., vol. 92, No. A9, 1987, p. 10101.

5. Wilson, R.M., Hathaway, D.H., and Reichmann, E.J.: “On the Importance of Cycle Minimum in Sunspot Cycle Prediction.” NASA Technical Paper 3648, Marshall Space Flight Center, AL, August 1996, 16 pp.

6. Hathaway, D.H., Wilson, R.M., and Reichmann, E.J.: Solar Phys., vol. 151, No. 1, 1994, p. 177.

7. Kopecký, M.: Bull Astron. Inst. Czech., vol. 42, 1991, p. 157.

8. Wilson, R.M.: Solar Phys., vol. 140, No. 1, 1992, p. 181.

9. Wilson, R.M., Hathaway, D.H., and Reichmann, E.J.: “On Determining the Rise, Size, and Duration Classes of a Sunspot Cycle.” NASA Technical Paper 3652, Marshall Space Flight Center, AL, September 1996, 14 pp.

10. Wilson, R.M.: Solar Phys., vol. 125, No. 1, 1990, p. 143.

11. Everitt, B.S.: “The Analysis of Contingency Tables.” John Wiley and Sons, New York, 1977.

12. Langley, R.: “Practical Statistics Simply Explained.” Revised edition, Dover Publishing, Inc., New York, 1971.

13. Lapin, L.L.: “Statistics for Modern Business Decisions.” Second edition, Harcourt Brace Jovanovich, Inc., New York, 1978.

14. Kane, R.P.: Nature, vol. 274, 1978, p. 139.

15. Kane, R.P.: Solar Phys., vol. 108, No. 2, 1987, p. 415.

16. Ohl, A.I.: Solar Dann., No. 12, 1966, p. 84.

17. Ohl, A.I., and Ohl, G.I.: in “Solar-Terrestrial Predictions Proceedings,” R.F. Donnelly (ed.), NOAA, Boulder, CO, 1979, p. 258.

18. Ehrenberg, A.S.C.: “A Primer in Data Reduction: An Introductory Statistics Textbook.” John Wiley and Sons, New York, 1982.

19. Fyodorov, M.V., Klimenko, V.V., and Dovgalyuk, V.V.: Solar Phys., vol. 165, No. 1, 1996, p. 193.

20. Schatten, K.H., and Pesnell, W.D.: Geophys. Res. Lett., vol. 20, No. 20, 1993, p. 2275.

21. Schatten, K.H., Myers, D.J., and Sofia, S.: Geophys. Res. Lett., vol. 23, No. 6, 1996, p. 605.

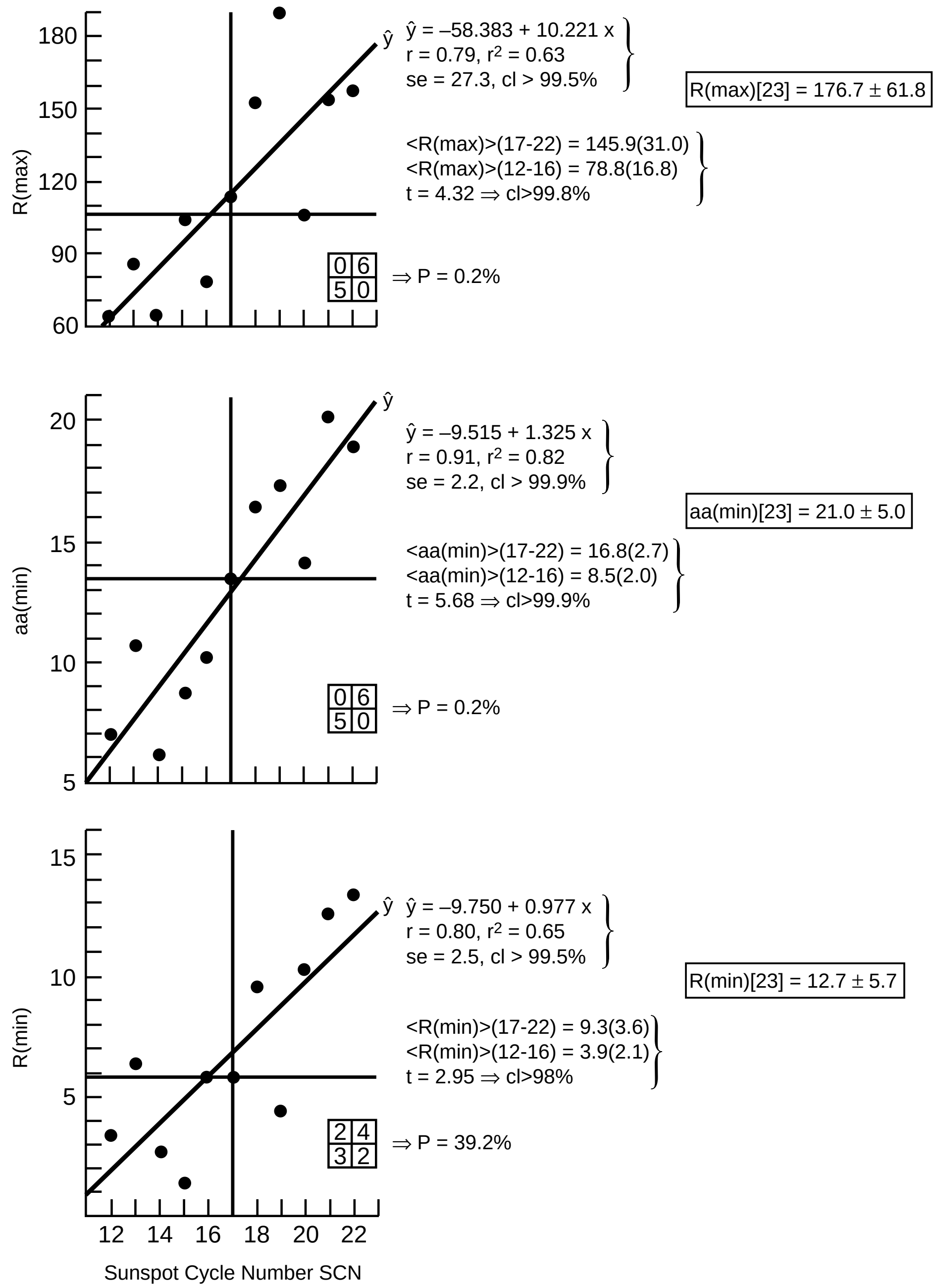

Figure 1. The variation of R(min), aa(min), and R(max) against sunspot cycle number (SCN) for the common data-available interval of cycles 12 to 22. The vertical and horizontal lines are median values of the parameters. The diagonal line is the regression fit, suggesting that each has embedded within it a long-term, linear, secular increase. Identified are the regression ($\hat{y}$), the coefficient of correlation (*r*), the coefficient of determination (*r*-squared), the standard error of estimate (*se*), and the confidence level for the fit (*cl*). Also shown are the results of hypothesis testing based on the t statistic for independent samples for the sample means computed for cycles 12 to 16 and 17 to 22 (standard deviations are shown within the parentheses). The probability *P* of obtaining the observed 2 by 2 contingency tables, based on the medians, or ones more suggestive of a departure from independence (chance), are shown. When $P \leq 5$ percent, the result is considered statistically significant. When $cl \geq 95$ percent, the result is also considered statistically important. Finally, the 95-percent level of confidence prediction intervals for each parameter are given, based on the extrapolation of the regression fits to cycle 23.

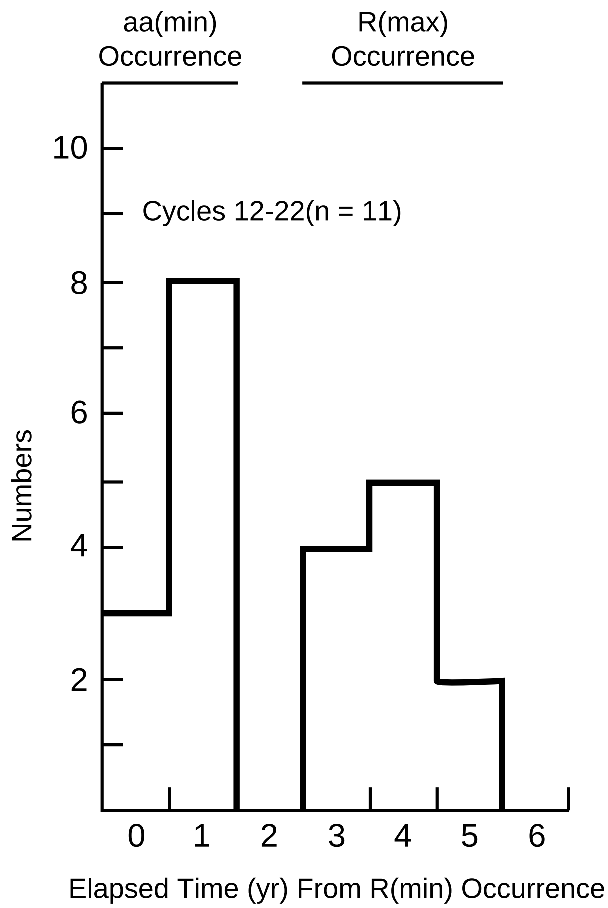

Figure 2. Frequency of occurrence histograms of aa(min) occurrences and R(max) occurrences relative to R(min) occurrences for cycles 12 to 22.

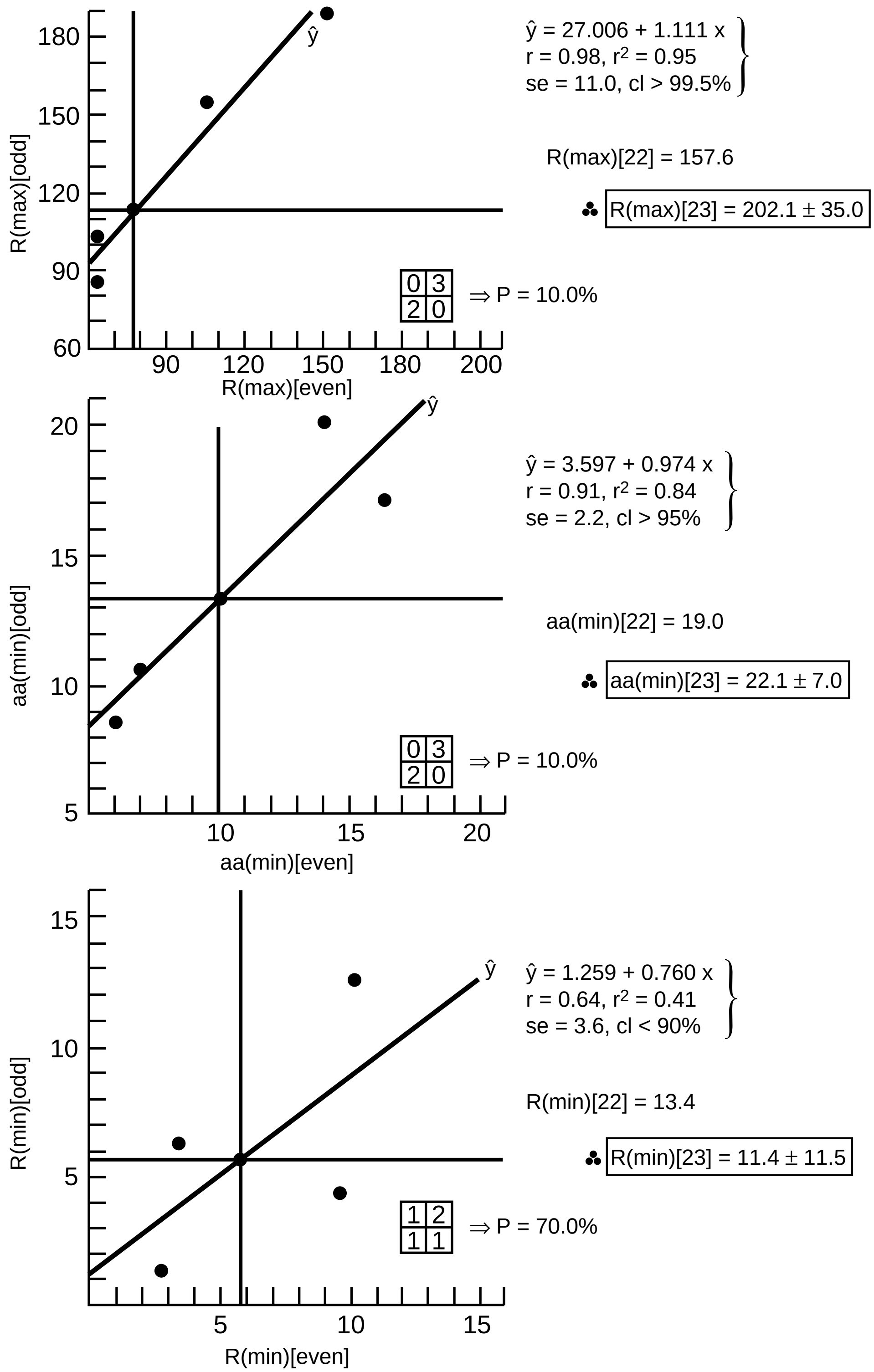

Figure 3. The even-odd cycle effect for R(min), aa(min), and R(max). The construction and interpretation follows that described in figure 1. The values of the parameters are shown for cycle 22 and the estimated values for cycle 23 are given, based on the cycle 22 measurements.

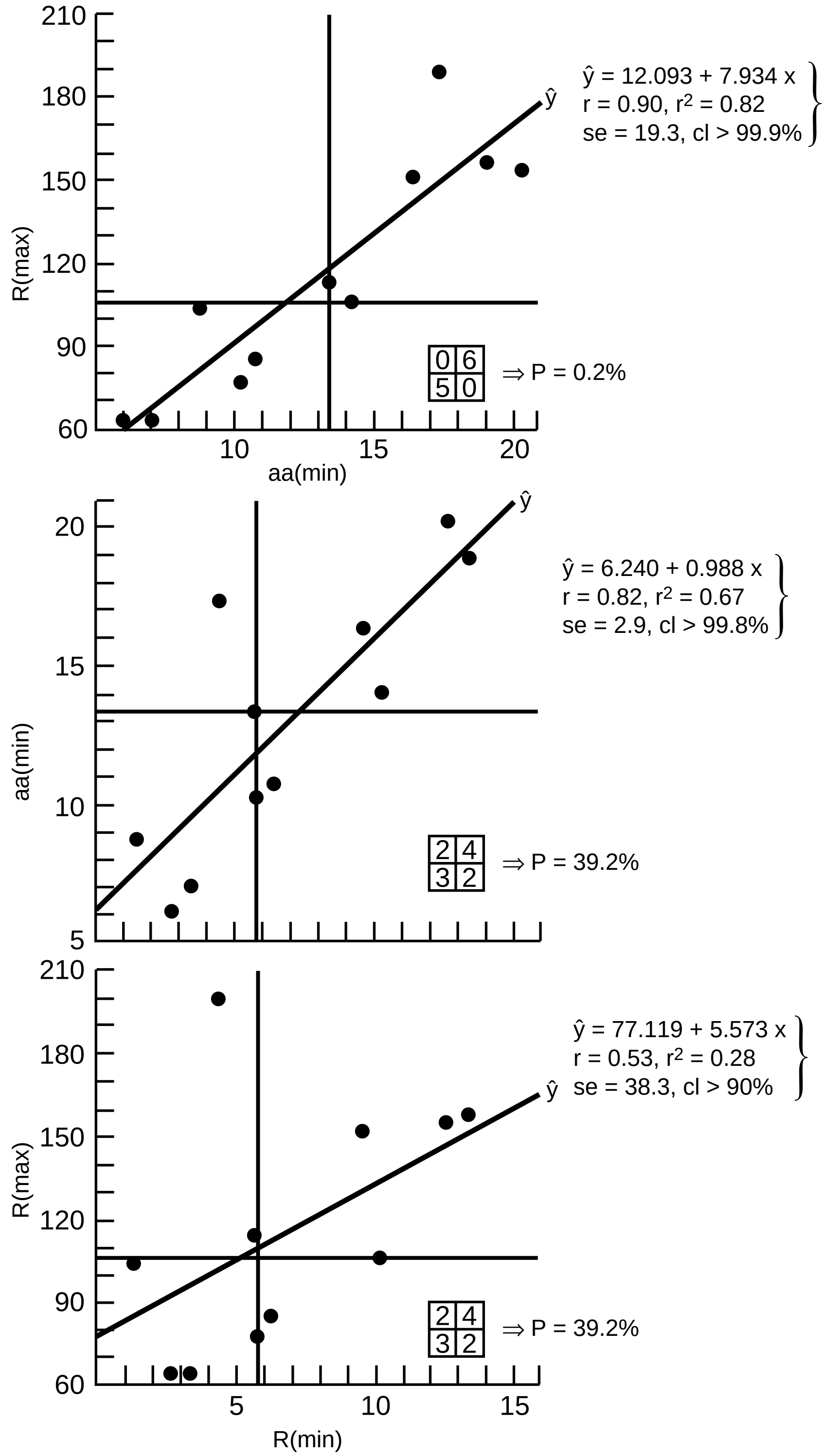

Figure 4. Correlational aspects of R(min), aa(min), and R(max). Construction and interpretation follows that described in figure 1.

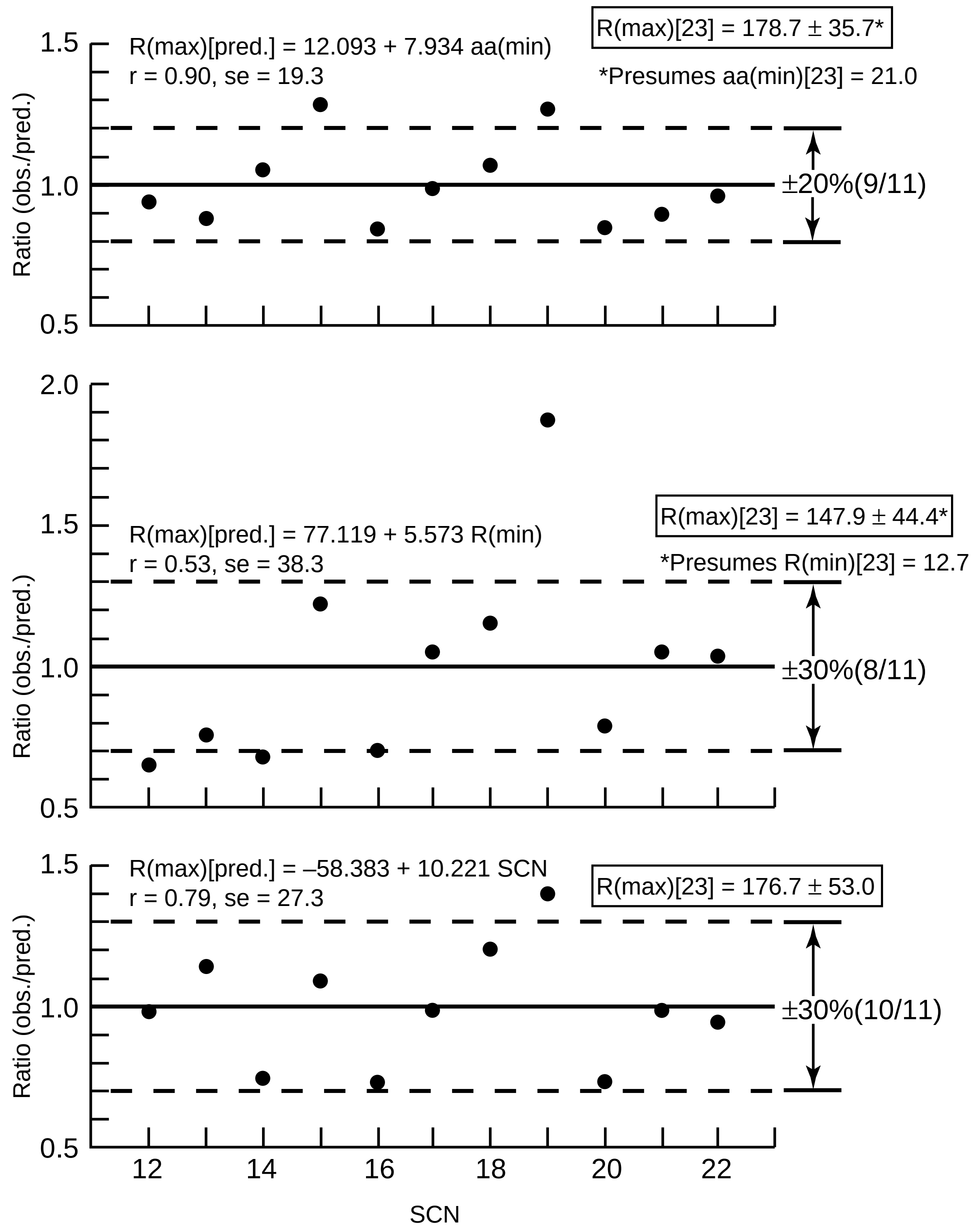

Figure 5. The relative error between observed value and predicted value for R(max) based on the secular fit (bottom), R(min) (middle), and aa(min) (top). The relative error is plotted as a ratio: observed R(max) divided by predicted R(max). The regression equation, *r*, and *se* are shown for each correlation. The interval in which the bulk of cycles 12 to 22 were found is identified as a percentage, with the fraction of cycles occurring within the interval shown in parentheses. The prediction intervals of R(max) for cycle 23 are shown, based on the regression fits and the identified percentages. For the R(max) versus R(min) fit, an R(min) = 12.7 was used to forecast R(max). For the R(max) versus aa(min) fit, an aa(min) = 21.0 was used to forecast R(max).

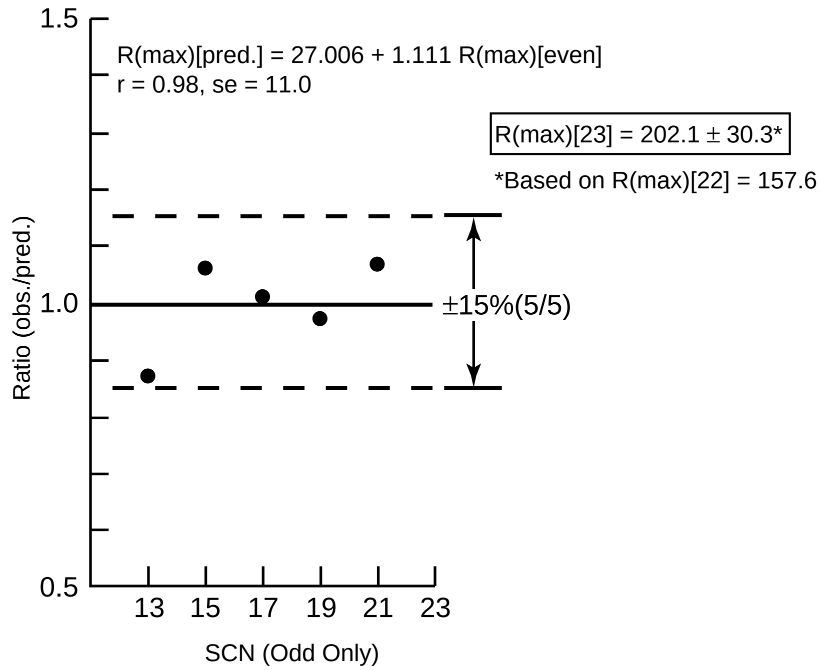

Figure 6. The relative error as a ratio for the even-odd fit. The R(max) for cycle 23 is based on an R(max) of 157.6 for cycle 22.

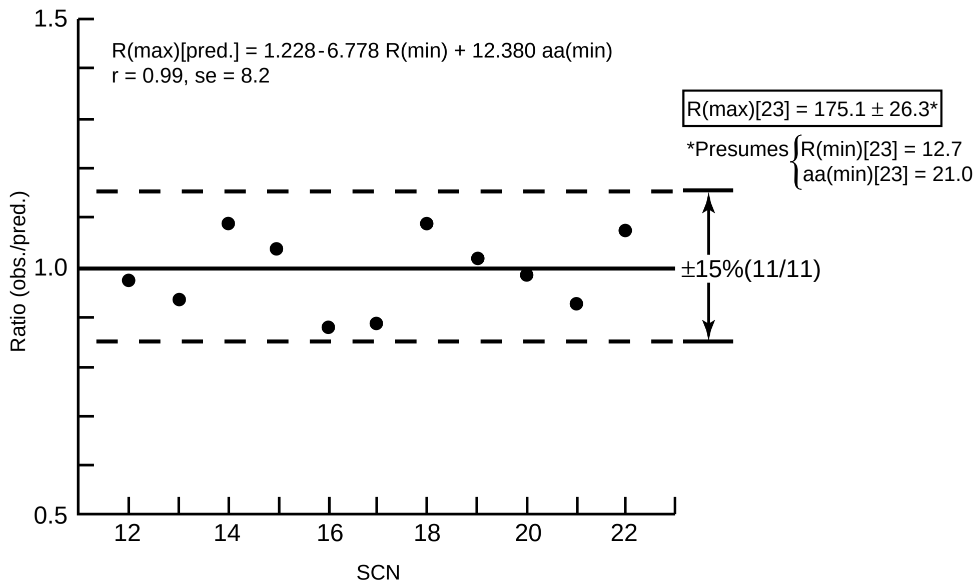

Figure 7. The relative error as a ratio for the bivariate fit, based on R(min) and aa(min). The R(max) for cycle 23 is based on an R(min) = 12.7 and an aa(min) = 21.0 for cycle 23, obtained from the secular fit.

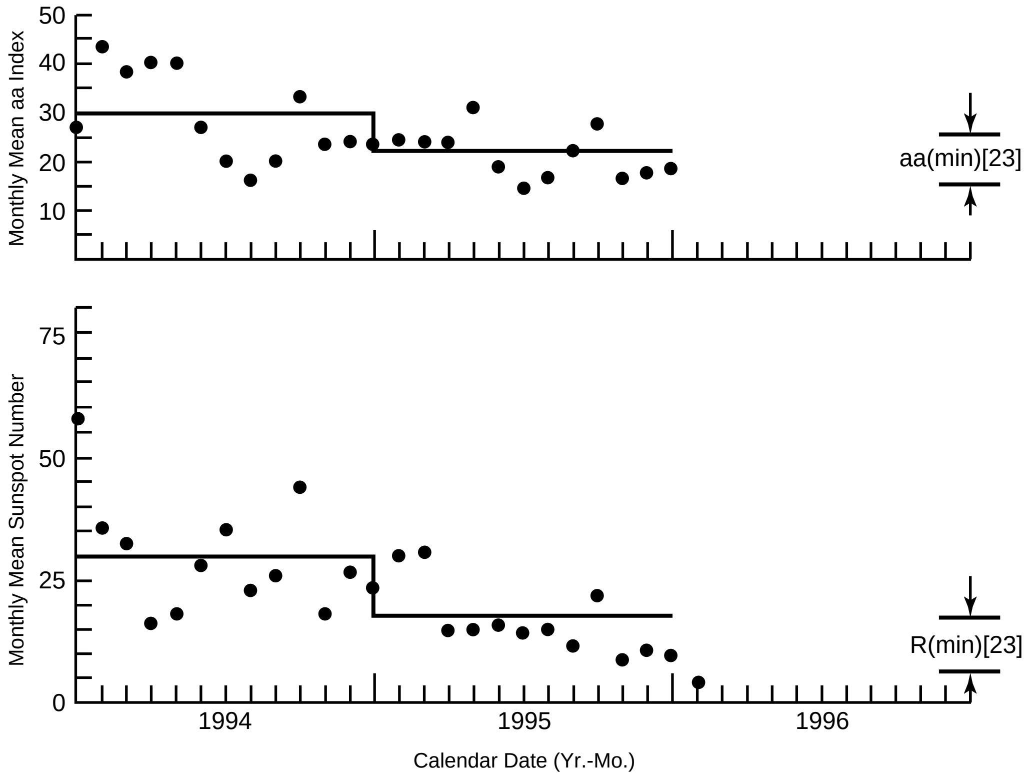

Figure 8. The monthly mean values of the aa geomagnetic index (top) and sunspot number (bottom). Annual averages are shown as horizontal lines. The expected values for aa(min) and R(min) for cycle 23 are shown at the right, based on the 95-percent level of confidence intervals of the secular fits.

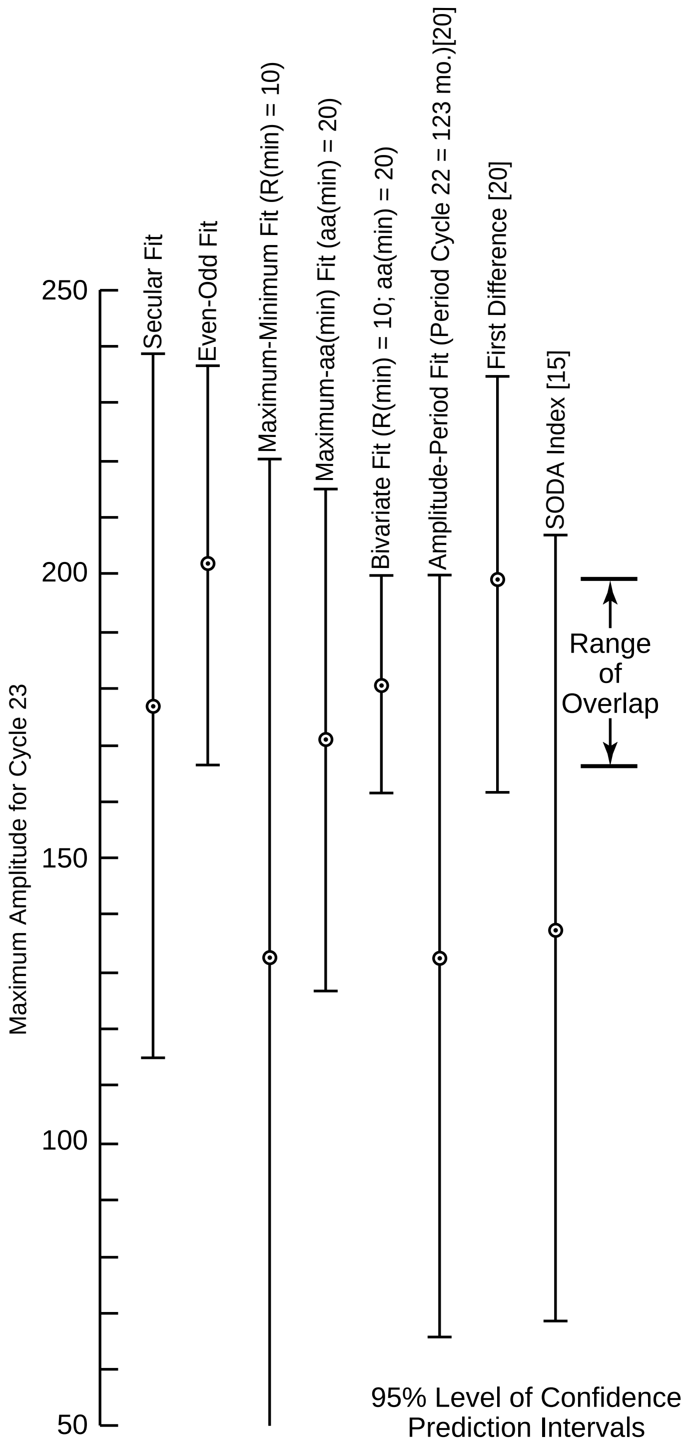

Figure 9. A comparison of maximum amplitude estimates for cycle 23. The maximum amplitude shown is an annual average for all the fits identified, except the amplitude-period fit and those based on the first difference and the SODA index, which refer to smoothed sunspot number maximum amplitudes and differ from the annual value by a very small amount. The overlap of the prediction intervals of R(max) for cycle 23 is about 167 to 200.